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ỨNG DỤNG BỘ ĐIỀU KHIỂN KHỚP NỐI MÔ HÌNH TIỂU NÃO CẢM XÚC NÃO MỜ (FBECMAC) CHO ĐIỀU KHIỂN TÁCH KÊNH HỆ PHI TUYẾN SIMO

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Tóm tắt: Bài báo này trình bày bộ điều khiển khớp nối mô hình tiểu não cảm xúc não mờ để điều khiển tách rời một lớp hệ thống phi tuyến thiếu điều khiển, đơn đầu vào, đa đầu ra. Trước tiên, một khung điều khiển trượt tách rời được xây dựng để chuyển đổi mục tiêu điều khiển thành ổn định bề mặt sai số ghép nối, và sau đó FBECMAC được sử dụng để xấp xỉ luật điều khiển lý tưởng. Một bộ bù mạnh mẽ phụ trợ được giới thiệu để triệt tiêu sai số xấp xỉ dư và duy trì ổn định vòng kín. Trái ngược với những tuyên bố không nhất quán trong các bản thảo trước đây, hiệu quả của phương pháp đề xuất chỉ được đánh giá trên hệ thống cầu-thanh phi tuyến được xem xét trong bài báo này. Kết quả mô phỏng cho thấy bộ điều khiển đề xuất ổn định đồng thời góc thanh và vị trí cầu, giảm sai số theo dõi và mang lại tác động điều khiển mượt mà hơn so với bộ điều khiển trượt thông thường.

Từ khóa: *Hệ thống tách rời thiếu điều khiển, Hệ thống mờ, Bộ điều khiển khớp nối mô hình tiểu não, Bộ điều khiển học tập cảm xúc não, Cần cẩu cầu.*

APPLICATIONS OF FUZZY BRAIN EMOTIONAL CEREBELLAR MODEL ARTICULATION CONTROLLER FOR SIMO NONLINEAR SYSTEM DECOUPLING CONTROL

Abstract: This paper presents a Fuzzy Brain Emotional Cerebellar Model Articulation Controller (FBECMAC) for decoupling control of a class of single-input multi-output underactuated nonlinear systems. A decoupled sliding-mode framework is first constructed to transform the control objective into coupled error-surface stabilization, and the FBECMAC is then employed to approximate the ideal control law. An auxiliary robust compensator is introduced to suppress the residual approximation error and preserve closed-loop stability. In contrast to the earlier inconsistent manuscript statements, the effectiveness of the proposed method is evaluated only on the nonlinear ball-beam system considered in this paper. The simulation results show that the proposed controller stabilizes the beam angle and ball position simultaneously, reduces tracking error, and yields smoother control action than the conventional sliding-mode controller.

Keywords: *Underactuated decoupling system, fuzzy system, cerebellar model articulation controller (CMAC), brain emotional learning controller (BELC), ball-beam system.*

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1. Introduction

In recent years, sliding-mode control (SMC) has been suggested as an approach for the control of systems with nonlinearities, uncertain dynamics and bounded input disturbances. The most popular conventional control approach that can be used for the control of such mechanisms is the robust control approach (Utkin, 1992; Åström and Wittenmark, 1995). The decoupled sliding-mode control is designed for a class of underactuated nonlinear systems. Each subsystem, which is decoupled into two second-order systems, is said to have main and sub-control purpose. Two sliding surfaces are constructed through the state variables of the decoupled subsystem. We define main and sub-target condition for these sliding surfaces, and introduce an intermediate variable from the sub-sliding surface condition (Hung and Chung, 2007; Lin and Mon, 2005).

A fuzzy inference system mimics the human reasoning process and is widely used successfully in various fields. Based on a biological prototype of the human brain and improved understanding of the functionality of the neurons and the pattern of their interconnections in the brain, a theoretical model used to explain the information-processing characteristics of the cerebellum was developed by Marr (1969). The brain has an orbitofrontal cortex and an amygdala; the former is a sensory neural network and the latter is an emotional neural network (Jan, 2002). These two networks affect each other, and the output of Brain Emotional Cerebellar Model Articulation Controller (BECMAC) contains two networks (Chung, 2014).

In Zheng (2017), the cerebellar model articulation controller has been used to replace the orbitofrontal cortex of brain. This change can accelerate the convergence time. At the same time, by using the fuzzy inference system into a BECMAC, a fuzzy neural network referred to as a fuzzy BECMAC (FBECMAC) has been proposed.

In this paper, a decoupling sliding-mode control (DSMC) strategy based on FBECMAC is developed for a SIMO underactuated nonlinear system. The proposed controller consists of a primary FBECMAC approximator and an auxiliary robust compensator. The FBECMAC is designed to emulate the ideal control law, whereas the auxiliary term is introduced to attenuate the approximation error and strengthen closed-loop robustness.

The proposed algorithm is validated through numerical simulation of a nonlinear ball-beam system only. Therefore, all discussions in the abstract, simulation section, and conclusion are consistently restricted to the ball-beam example.

2. Mathematical model of FBECMAC

An FBECMAC can be classified as a supervised network, and it is not very complex in operation and has fast convergence property, so it is applicable to many nonlinear control systems. An FBECMAC has two systems; the first system is an amygdala system, and the second system is a cerebellar system, and they imitate to that in a mammalian brain.

The FBECMAC has the following form:

Based on the FBECMAC structure, the FCMAC network is proposed using the fuzzy inference rules:

$$\text{If } I_1 \text{ is } J_{1jk},\ I_2 \text{ is } J_{2jk},\ldots, \text{ and } I_{n_i} \text{ is } J_{n_i jk} \text{ then}$$

$$a_o \text{ is } w_{jko} \text{ for } j=1,2,3,\ldots,n_j, k=1,2,3,\ldots,n_k, o=1,2,3,\ldots,n_o$$

where I_i is the input of fuzzy inference system, w_{jko} is the FCMAC weight and a_o is the FCMAC output.

The signal propagation and the basic function in each space of FCMAC are introduced as follows:

2.1. Input space

For a given $I = [I_1, I_2, \dots, I_{n_i}] \in \Re^{n_i}$, where n_i represent the ith input to the node of layer 1. Each input state variable I can be quantized into discrete regions (call an element) according to a given control space. The number of elements is referred to as a resolution.

2.2. Association memory space

Several elements can be accumulated as a block. In this space each block acts as a receptivefield basis function. The Gaussian function is adopted here as receptive-field basis function, which can be represented as:

$$f_{ijk}(F_{ijk}) = \exp(-F_{ijk}{}^2) \\ for\ i = 1, 2, \dots, n_i,\ j = 1, 2, \dots, n_j, and\ i = 1, 2, \dots, n_i \tag{1}$$

Here, $F_{ijk} = \dfrac{(I_i - m_{ijk})^2}{\sigma_{ijk}^2}$, where m_{ijk} is a mean parameter, and σ_{ijk} is a variance parameter of the Gaussian function.

2.3. Receptive-field space

The multidimensional receptive field function is defined as:

$$r_{jk} = \prod_{i=1}^{n_i} f_{ijk}(F_{ijk}) = \prod_{i=1}^{n_i}\left(\exp\left(-\frac{(I_i - m_{ijk})^2}{\sigma_{ijk}^2}\right)\right) \\ for\ j = 1, 2, \dots, n_j, and\ k = 1, 2, \dots, n_k \tag{2}$$

where r_{jk} is associated with the jth layer and kth block.

The multidimensional receptive-field functions can be expressed in a vector form as:

$$r = [r_{11}, \dots, r_{1n_k}, r_{21}, \dots, r_{2n_k}, \dots r_{n_j 1}, \dots, r_{n_j n_k}]^T \in \Re^{n_j n_k} \tag{3}$$

2.4. Weight memory space

Each location of hypercube to a particular adjustable value in the weight memory space can be expressed as:

$$w = \begin{bmatrix} w_{111} & w_{112} & \cdots & w_{11n_o} \\ \vdots & \vdots & & \vdots \\ w_{1n_k 1} & w_{1n_k 2} & \cdots & w_{1n_k n_o} \\ \vdots & \vdots & & \vdots \\ w_{n_j n_k 1} & w_{n_j n_k 2} & \cdots & w_{n_j n_k n_o} \end{bmatrix} \in \Re^{n_j n_k \times n_o} \tag{4}$$

where w_{jko} denotes the connecting weight value of the output associated with the jth layer, kth block and o is the output number.

2.5. FCMAC output space

The output of FCMAC is the algebraic sum of the activated weights in the weight memory and the o th output is expressed as:

$$u_{FCMACo} = a_o = w^T r = \sum_{j=1}^{n_j} \sum_{k=1}^{n_k} r_{jk} w_{jko} \quad for\ o = 1, 2, \ldots, n_o \tag{5}$$

Based on the FBECMAC structure, the FBELC network is proposed using the fuzzy inference rules:

If I_1 is λ_{1j}, I_2 is λ_{2j} ,…, and I_{n_i} is $\lambda_{n_i j}$ then

o_o is v_{ijo} for $i = 1, 2, 3, \ldots, n_i$, $j = 1, 2, 3, \ldots, n_j$, $o = 1, 2, 3, \ldots, n_o$

where I_i is the input of fuzzy inference system, v_{jko} is the emotional weight and o_o is the FBELC output.

The signal propagation and the basic function in each space of FBELC are introduced as follows:

2.6. Input space

Given the input data I_i , $i = 1, 2, \ldots, n_i$, each input state variable I_i is assumed to be quantized into n_e discrete regions (called "elements" or "neurons") according to a given control space. The number of elements n_e is termed as a resolution. Based on the input variable I_i , totally j layers are defined.

2.7. Sensory cortex space

In this space, the sigmoid function is adopted represented as:

$$\lambda_{ij} = \exp\left(\frac{-(I_i - m_{ij})^2}{\sigma_{ij}^2} \right) \tag{6}$$

where λ_{ij} is the Gaussian function of prefrontal system input and amygdala system input for the sensory cortex output, I_i is the system input and m_{ij} is the mean and σ_{ij} is the variance.

2.8. Emotional weight space

Each sensory cortex output to a particular adjustable value in the emotional weight space can be expressed as:

$$v = \begin{bmatrix} v_{111} & v_{112} & \cdots & v_{11n_o} \\ \vdots & \vdots & & \vdots \\ v_{1n_j1} & v_{1n_j2} & \cdots & v_{1n_jn_o} \\ \vdots & \vdots & & \vdots \\ v_{n_in_j1} & v_{n_in_j2} & \cdots & v_{n_jn_jn_o} \end{bmatrix} \in \Re^{n_i n_j \times n_o} \tag{7}$$

where v_{ijo} denotes the connecting weight of the output associated with the ith input, jth layer and oth output.

2.9. FBELC output space

The o-th output of FBELC is represented as:

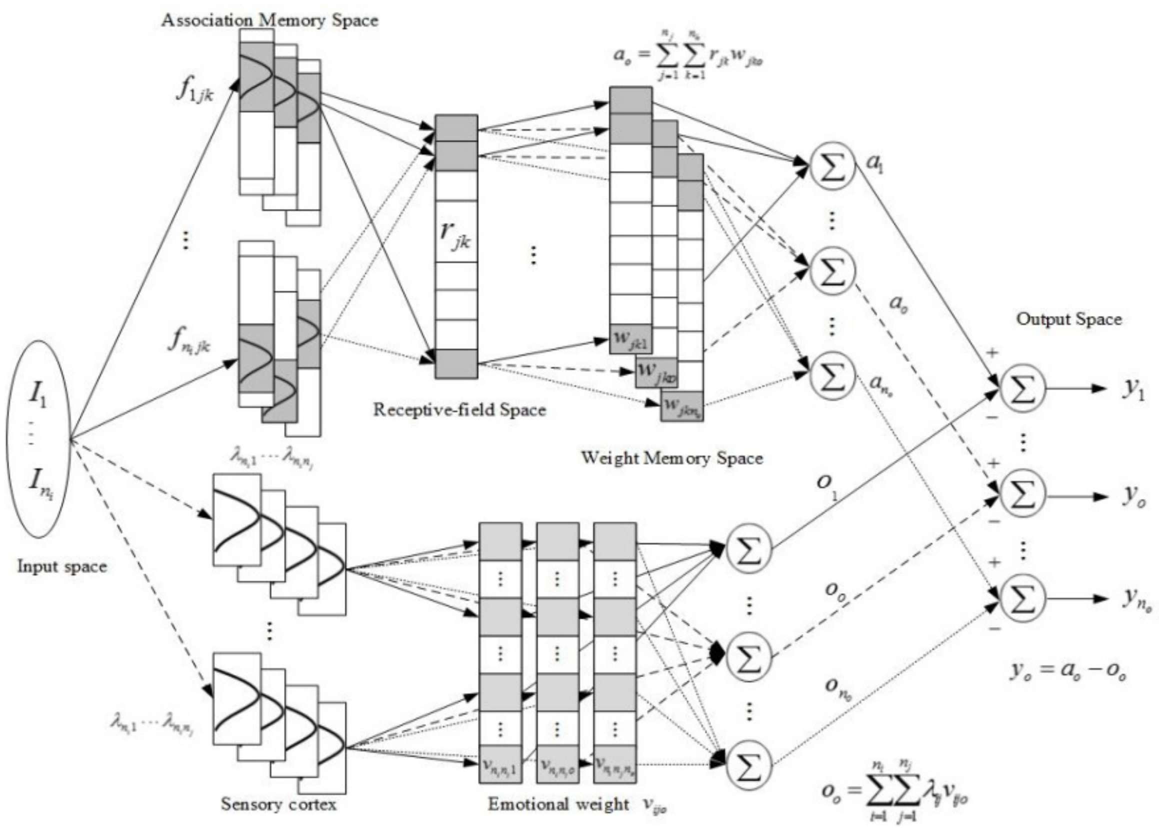
$$u_{FBELC_o} = o_o = \sum_i \sum_j \lambda_{ij} v_{ijo} \quad \text{for } i = 1,2,\cdots,n_i,\ j = 1,2,\cdots,n_j, o = 1,2,\ldots,n_o \tag{8}$$

The total o th output of FBECMAC is given as:

$$y_o = a_o - o_o = u_{FCMAC_o} - u_{FBELC_o} = u_{FBECMAC_o} \tag{9}$$

The FBECMAC is proposed as shown in Fig. 1.

Figure 1. Brain Emotional Cerebellar Model Articulation controller



3. Problem Formulation

Consider a single-input–multi-output (SIMO) underactuated nonlinear coupled system expressed in the following form:

$$\begin{aligned} \dot{x}_1(t) &= x_2(t) \\ \dot{x}_2(t) &= g_1(x_1, x_2) + b_1(x_1, x_2)u(t) + d_1(t) \\ \dot{x}_3(t) &= x_4(t) \\ \dot{x}_4(t) &= g_2(x_1, x_2) + b_2(x_1, x_2)u(t) + d_2(t) \end{aligned} \tag{10}$$

where $x=[x_1, x_2, x_3, x_4]^T$ denotes the system state vector, $y=[y_1, y_2]^T$ is the measured output vector, $y_d=[y_{d1}, y_{d2}]^T$ is the reference vector, $e = y_d - y$ is the tracking-error vector, s denotes the coupled sliding surface, u is the scalar control input, and d_1, d_2 represent lumped uncertainties and external disturbances. All vectors are assumed to be dimensionally compatible, and all scalar design constants are chosen to be positive unless otherwise stated.

This system can be treated as two subsystems with second-order canonical form including the states (x_1, x_2) and (x_3, x_4), respectively. The decoupling control tries to design a single input u to simultaneously control the states (x_1, x_2) and (x_3, x_4) to achieve desired performance.

3.1. Sliding Mode Control

Define the tracking error as:

$$E(t) = X_d(t) - X(t) \in \Re^n \tag{11}$$

Where $X_d(t)=\begin{bmatrix} x_{1d}(t) & x_{2d}(t) & x_{3d}(t) & x_{4d}(t)\end{bmatrix}^T \in \Re^4$

and $X(t)=\begin{bmatrix} x_1(t) & x_2(t) & x_3(t) & x_4(t)\end{bmatrix}^T \in \Re^4$ denote the reference trajectory and output vector of the system, respectively; and $e_i = x_{id} - x_i$ for $i=1, 2, 3, 4$.

In (10) and (11), we can define the coupling sliding surface for this system as reported by Hung and Chung (2007).

$$s_1 = c_1(e_1 - z) + e_2 \tag{12}$$

$$s_2 = c_1 e_3 + e_4 \tag{13}$$

where z is derived from s_2 and is defined as:

$$z = \mathrm{sat}(s_2/\Phi_z)\, z_u, \quad 0 < z_u < 1 \tag{14}$$

where Φ_z is the boundary layer of s_2. To smooth out z, Φ_z transfers s_2 to the proper range of x_1, and the definition of sat(.) function is:

$$sat(\varphi) = \begin{cases} \mathrm{sgn}(\varphi), & if \ |\varphi| \geq 1 \\ \varphi, & if \ |\varphi| < 1 \end{cases} \tag{15}$$

Notice that z is a decaying oscillation signal because z_u is a factor less than one.

Consider (12). If $\lim_{t\to\infty} s_1(t) \to 0$, then $\lim_{t\to\infty} e_1(t) \to z$, $\lim_{t\to\infty} e_2(t) \to 0$. Since z is a value transferred from s_2, when $\lim_{t\to\infty} s_2(t) \to 0$, then $\lim_{t\to\infty} z(t) \to 0$ and $\lim_{t\to\infty} e_1(t) \to 0$ with (13), if the condition $\lim_{t\to\infty} s_1 \to 0$, the control objective is achieved. The choice of c_1 and c_2 has strong influence on the behavior in the transient state of the system. The appropriate choice is necessary for achieving favorable transient response. The sliding mode control law is given as:

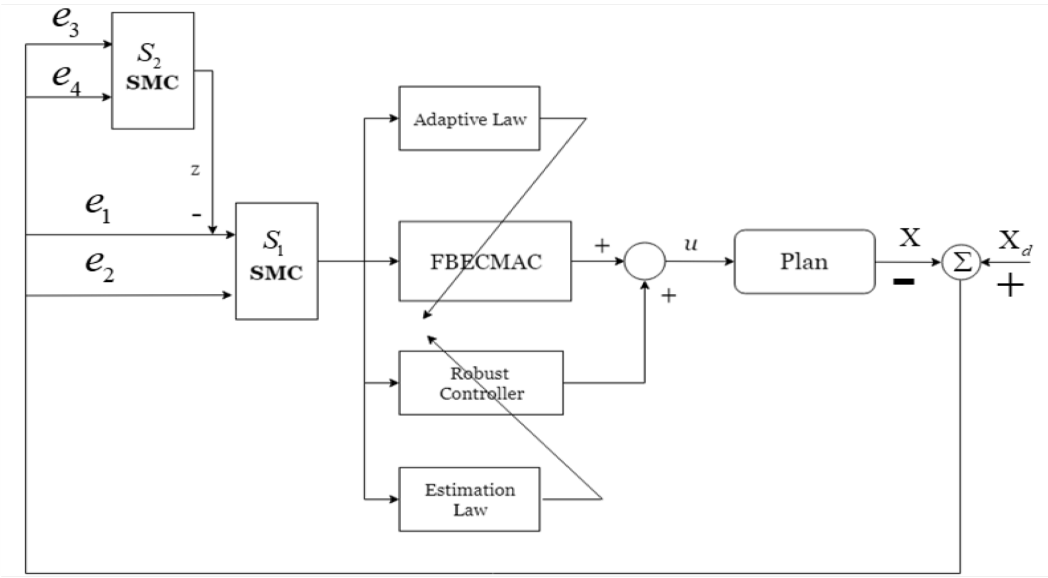
$$u(t) = u_{FBELCMAC}(t) + u_r(t) \tag{16}$$

where the ideal controller $u_I(t)$ is represented as:

$$u_I = b_1^{-1}\left[\text{-}cx_2 - c\dot{z} - g_1 + c\dot{x}_{1d} + \dot{x}_{2d} - d_1\right] \tag{17}$$

u_I in (17) is unavailable. Thus, an FBECMAC is proposed to approach the ideal controller.

Figure 2. Block diagram of FBECMAC control system using a robust compensator



3.2. FBECMAC and Robust Control System Design

Since the FBECMAC cannot completely imitate the ideal controller, the approximation error will induce the tracking error of the control system; thus a robust controller is needed to achieve the robust stability of the control system.

Taking the derivative of (12) and using (16), yields:

$$\begin{aligned} \dot{s}_1(t) &= c_1(\dot{e}_1 - \dot{z}) + \dot{e}_2 \\ &= c_1\left(\dot{x}_{1d} - x_2 - \dot{z}\right) + \dot{x}_{2d} - \dot{x}_2 \end{aligned} \tag{18}$$

If $\frac{1}{2} s_1^T(t) s_1(t)$ is chosen as a cost function, then its derivative is $s_1^T(t)\dot{s}_1(t)$. The FBECMAC parameter adjusting laws is given in (19) to (24), where the training algorithms in (25) to (30) perform error back propagation, using the following chain-rule algorithm:

$$w_{jko}(t+1) = w_{jko}(t) + \Delta w_{jko} \tag{19}$$

$$m_{ijk}(t+1) = m_{ijk}(t) + \Delta m_{ijk} \tag{20}$$

$$\sigma_{ijk}(t+1) = \sigma_{ijk}(t) + \Delta \sigma_{ijk} \tag{21}$$

$$v_{ijo}(\mathrm{t+1}) = v_{ijo}(\mathrm{t}) + \Delta v_{ijo} \tag{22}$$

$$m_{ij}(t+1) = m_{ij}(t) + \Delta m_{ij} \tag{23}$$

$$\sigma_{ij}(t+1) = \sigma_{ij}(t) + \Delta \sigma_{ij} \tag{24}$$

$$\Delta w_{jko} = -\eta_w \frac{\partial s^T \dot{s}}{\partial w_{jko}} = -\eta_w \frac{\partial s^T \dot{s}}{\partial u_{FBECMAC_o}} \frac{\partial u_{FBECMAC_o}}{\partial w_{jko}} = \eta_w s_i r_{jk} \tag{25}$$

$$\begin{aligned} \Delta m_{ijk} &= -\eta_{mc} \frac{\partial s^T \dot{s}}{\partial m_{ijk}} = -\eta_{mc} \frac{\partial s^T \dot{s}}{\partial u_{FBECMAC_o}} \frac{\partial u_{FBECMAC_o}}{\partial r_{jk}} \frac{\partial r_{jk}}{\partial m_{ijk}} \\ &= \eta_{mc} s_i \sum_{o=1}^{n_o} w_{jko} r_{jk} \frac{2(I_i - m_{ijk})}{\sigma_{ijk}{}^2} \end{aligned} \tag{26}$$

$$\begin{aligned} \Delta s_{ijk} &= -\eta_{vc} \frac{\partial s^T \dot{s}}{\partial m_{ijk}} = -\eta_{vc} \frac{\partial s^T \dot{s}}{\partial u_{FBECMAC_o}} \frac{\partial u_{FBECMAC_o}}{\partial r_{jk}} \frac{\partial r_{jk}}{\partial v_{ijk}} \\ &= \eta_{vc} s_i \sum_{o=1}^{n_o} w_{jko} r_{jk} \frac{2(I_i - m_{ijk})^2}{\sigma_{ijk}{}^3} \end{aligned} \tag{27}$$

$$\Delta v_{ijo} = -\eta_V \frac{\partial s^T \dot{s}}{v_{ijo}} = -\eta_{vb} \frac{\partial s^T \dot{s}}{\partial u_{FBECMAC_o}} \frac{\partial u_{FBECMAC_o}}{\partial v_{ijo}} = -\eta_{vb} s_i \lambda_{ij} \tag{28}$$

$$\begin{aligned} \Delta m_{ij} &= -\eta_{mb} \frac{\partial s^T \dot{s}}{\partial m_{ij}} = -\eta_{mb} \frac{\partial s^T \dot{s}}{\partial u_{FBECMAC_o}} \frac{\partial u_{FBECMAC_o}}{\partial \lambda_{ij}} \frac{\partial \lambda_{ij}}{\partial m_{ij}} \\ &= -\eta_{mb} s_i \sum_{o=1}^{n_o} v_{ijo} \frac{2(I_i - m_{ij})}{\sigma_{ij}{}^2} \end{aligned} \tag{29}$$

$$\begin{aligned} \Delta \sigma_{ij} &= -\eta_{vb} \frac{\partial s^T \dot{s}}{\partial \sigma_{ij}} = -\eta_{vb} \frac{\partial s^T \dot{s}}{\partial u_{FBECMAC_o}} \frac{\partial u_{FBECMAC_o}}{\partial \lambda_{ij}} \frac{\partial \lambda_{ij}}{\partial \sigma_{ij}} \\ &= -\eta_{vb} s_i \sum_{o=1}^{n_o} v_{ijo} \frac{2(I_i - m_{ij})^2}{\sigma_{ij}{}^3} \end{aligned} \tag{30}$$

where s_i denotes the i-th component of s.

An approximation error between the FBECMAC and the ideal controller is unavoidable, so an ideal controller can be formulated as the summation of the FBECMAC and an approximation error, assume:

$$(u_I - u_{FBECMAC}) = \Delta \tag{31}$$

where denotes the approximation error. Based on the universal approximation theorem, a fuzzy inference system or a neural fuzzy system can approximate an unknown function with small approximation error (Wang, 1994; Lin and Lee, 1996).

In order to derive the decoupling sliding-mode controller with FNN based, substituting (16) into (10), we can obtain:

$$\begin{aligned} \dot{x}_2 &= f_1(x) + b_1(x)(u_{FBECMAC} + u_r) + d_1(t) \\ &= b_1(u_{FBECMAC} + u_r - u_I) + c_1 x_2 - c_1 \dot{z} + c_1 \dot{x}_{1d} + \dot{x}_{2d} \end{aligned} \tag{32}$$

Hence, (8) can be rewritten:

$$\dot{s}_1 = b_1 \left(u_I - u_{FBECMAC} - u_r \right) \tag{33}$$

so that a robust controller u_r can be developed to attenuate the effect of the approximation error between u_I and $u_{FBECMAC}$, to allow better L_2 tracking. Assume Δ_p exists and it satisfies L_2 bounded, then for a specified L_2 tracking performance:

$$\sum_{p=1}^{n_p} \int_0^T b_1 s_p^2(t)\, dt \le s_1^T(0) s_1(0) + \sum_{p=1}^{n_p} r_p^2 \int_0^T b_1 \Delta_p^2(t)\, dt \tag{34}$$

where s_p is the p-th element of s_1, Δ_p is the p-th element of Δ, r is a prescribed positive attenuation constant for $p = 1, 2, \ldots, n_p$, and $b_1(x)$ is positive, the robust controller is designed as:

$$u_r(t) = (2R^2)^{-1}(R^2 + I)\, s(t) \tag{35}$$

where $R = diag[r_p]$ and I is a unity matrix. Then the following theorem can be stated and proven.

Theorem 1: For the feedback system shown in Fig.2, if the control system is given as (16), where the FBECMAC is given in (9) and the robust controller is given in (35), then the robust stability in (34) can be guaranteed.

Proof: The Lyapunov function is given by:

$$V(s_1(t)) = \frac{1}{2} s_1^T(t) s_1(t) \tag{36}$$

Taking the derivative of the Lyapunov function and using (32), (33) and (34), yields:

$$\begin{aligned}
\dot{V}(s_1(t)) &= s_1(t)^T \dot{s}_1(t) \\
&= s_1(t)^T b_1(x)[\Delta(t) - u_r(t)] \\
&= s_1(t)^T b_1(x)[\Delta(t) - (2R^2)^{-1}(R^2 + I)\, s(t)] \\
&= s_1(t)^T b_1(x)\Delta(t) - s_1(t) b_1(x)(2R^2)^{-1}(R^2 + I)\, s_1(t)] \\
&= s_1(t)^T b_1(x)\Delta(t) - s_1(t)^T s_1(t) b_1(x)(2R^2)^{-1}(R^2) \\
&\quad - s_1(t)^T s_1(t) b_1(x)(2R^2)^{-1}(I)] \\
&= s_1(t)^T b_1(x)\{\Delta(t) \\
&\quad -[(2R^2)^{-1}(R^2)s_1(t)] - [(2R^2)^{-1}(I)s_1(t)]\} \\
&= \sum_{p=1}^{n_p} b_1[s_p(t)\Delta_p(t) - \frac{s_p^2(t)}{2} - \frac{s_p^2(t)}{2r_p^2}] \\
&= b_1 \sum_{p=1}^{n_p} [-\frac{s_p^2(t)}{2} - \frac{1}{2}[2s_p(t)\Delta_p(t) + (\frac{s_p(t)}{r_p})^2 + (r_p\Delta_p(t))^2] - \frac{(r_p\Delta_p(t))^2}{2}] \\
&= b_1 \sum_{p=1}^{n_p} \{-\frac{s_p^2(t)}{2} - \frac{1}{2}[\frac{s_p(t)}{r_p(t)} + (r_p\Delta_p(t))^2]^2 + \frac{(r_p\Delta_p(t))^2}{2}\} \\
&\le b_1 \sum_{p=1}^{n_p} [-\frac{s_p^2(t)}{2} + \frac{r_p^2 \Delta_p{}^2(t)}{2}]
\end{aligned} \tag{37}$$

If it is assumed that $d(t) \in L_2[0,T],\ \forall T \in [0,\infty]$ then integrating the above equation from $t = 0$ to $t = T$ yields:

$$V(T) - V(0) \le b_1 \sum_{p=1}^{n_p} [-\frac{1}{2}\int_0^T s_p^2(t)\, dt + \frac{r_p^2}{2}\int_0^T \Delta_p^2(t)\, dt] \tag{38}$$

Since $V(T) \ge 0$, the above inequality implies the following inequality:

$$\frac{1}{2} b_1 \sum_{p=1}^{n_p} \int_0^T s_p^2(t)\, dt \le V(0) + \frac{1}{2}\sum_{p=1}^{n_p} r_p^2 \int_0^T b_1 \Delta_p^2(t)\, dt \tag{39}$$

Using (16) the above inequality is equivalent to the following:

$$\sum_{p=1}^{n_p} \int_0^T b_1 s_p^2(t)\, dt \ \le\ s_1^T(0)s_1(0) + \sum_{p=1}^{n_p} r_p^2 \int_0^T b_1 \Delta_p^2(t)\, dt \tag{40}$$

4. Nonlinear Ball-Beam System Control Simulation Using the Proposed Method

Figure 3. The ball-beam system

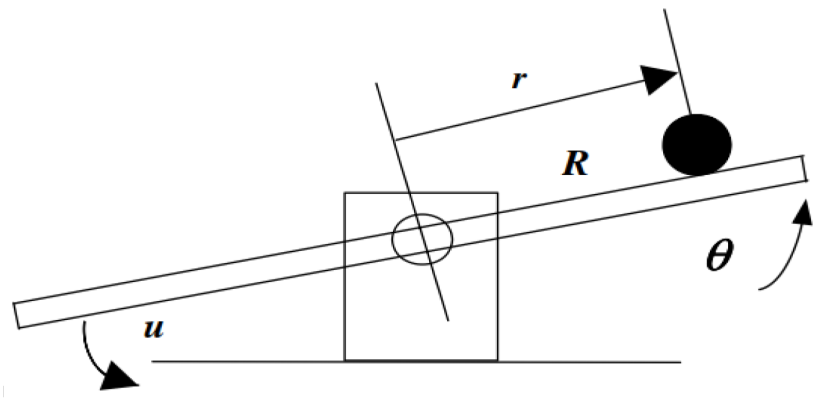


Table 3. the physical parameters of the ball-beam system

$x_1 = \theta$	the angle of the pole with respect to the vertical axis
$x_3 = r$	the position of the cart
$x_2 = \dot{\theta}$	the angle velocity of the pole with respect to the vertical axis
$x_4 = \dot{r}$	the velocity of the cart
M	the mass of the ball
J_b	moment of inertia of the ball
R	the radius of the ball
g	acceleration of gravity

In this section, the proposed controller is applied to the nonlinear ball-beam system in order to verify the theoretical development. The control objective is to regulate both the beam angle and the ball position to the equilibrium point while maintaining bounded control effort. Table 3 summarizes the physical parameters adopted in the simulation study. The dynamics of the ball-beam system shown in Fig. 3 can be expressed as follows:

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= u + d \\ \dot{x}_3 &= x_4 \\ \dot{x}_4 &= B\left(x_3 x_2^2 - G\sin x_1\right) \end{aligned} \tag{41}$$

The rotation center is assumed to be frictionless, allowing the ball to roll freely along the beam. It is necessary for the ball to maintain contact with the beam, with the condition that rolling occurs without slipping. The goal is to keep the ball near the center of the beam. In the following section, we introduce the relevant variables:

$$\begin{aligned} s_1 &= c_1(\theta - z) + \dot{\theta} = c_1(x_1 - z) + x_2 \\ s_2 &= c_2 x + \dot{x} = c_2 x_3 + x_4 \end{aligned} \tag{42}$$

and:

$$z = sat(s_2 / \Phi_z) \cdot Z_{upper} \text{, where } 0 < Z_{upper} < 1 \tag{43}$$

The simulation is conducted using the following parameters:

The initial conditions are selected as shown above. For comparison, both the conventional sliding-mode controller and the proposed FBECMAC-based controller are implemented under the same simulation conditions. Figures 4-6 indicate that both controllers stabilize the system; however, the proposed method provides faster convergence of the beam angle and ball position, smaller residual oscillation near the equilibrium point, and a smoother control signal. To make the comparison more rigorous, quantitative indices such as RMSE, settling time, maximum absolute control input, and integral absolute error (IAE) should be reported from the simulation data and summarized in Table 4.

Table 4. Quantitative comparison between the conventional SMC and the proposed FBECMAC controller

Performance index	SMC	Proposed FBECMAC	Remark
RMSE of beam angle	0.2031	0.1870	Lower is better
RMSE of ball position	2.6980	2.119	Lower is better
Settling time (2% band, both states)	5.45 s	6.62 s	Shorter is better
Maximum |u|	8.1087	10.0000	Smaller is smoother
IAE = $\int(\lvert e_q \rvert + \lvert e_x \rvert)dt$	23.0772	20.4052	Lower is better

Figure 4. Angle evolution of the beam

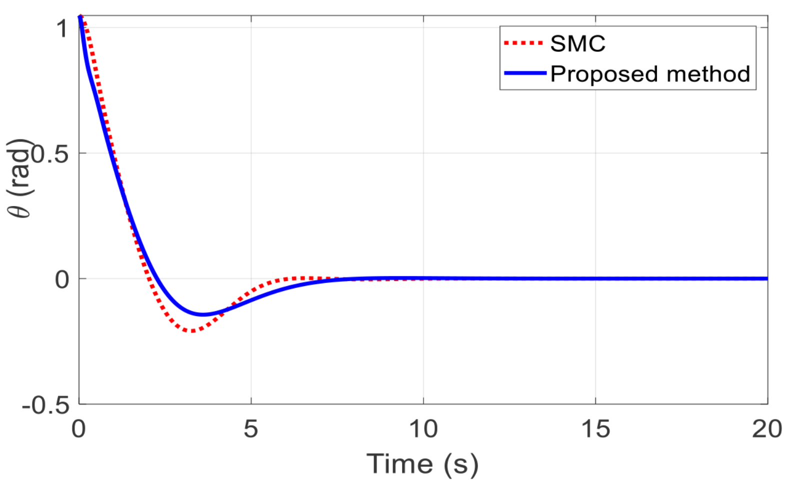


Figure 5. Position evolution X of the ball Figure

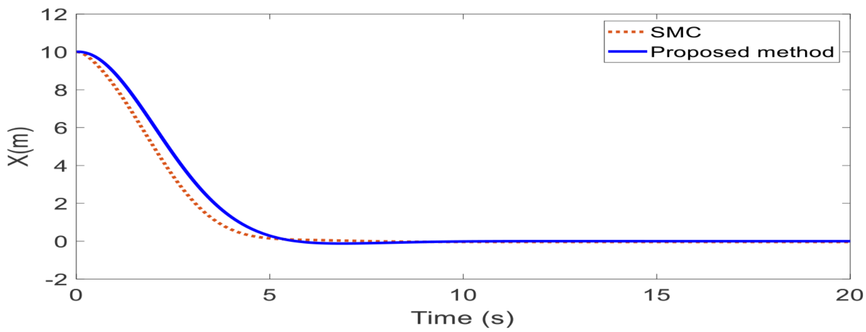


Figure 6. The control effort of the control system using SMC and proposed method

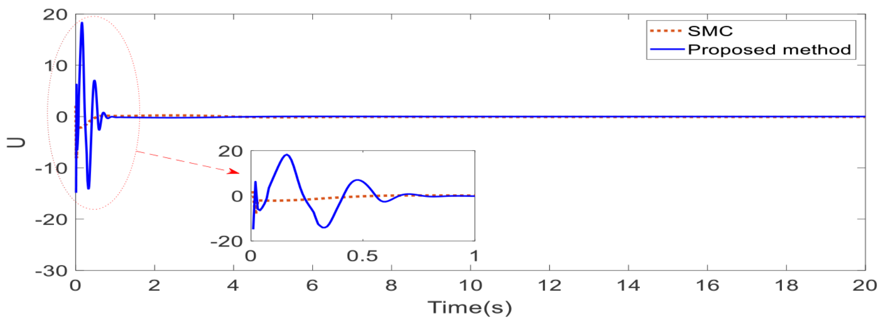


Table 5. Summary of abbreviations used in this paper

Abbreviation	Full term
SMC	Sliding-Mode Control
DSMC	Decoupling Sliding-Mode Control
SIMO	Single-Input Multi-Output
FBECMAC	Fuzzy Brain Emotional Cerebellar Model Articulation Controller
BECMAC	Brain Emotional Cerebellar Model Articulation Controller
FCMAC	Fuzzy Cerebellar Model Articulation Controller
FBELC	Fuzzy Brain Emotional Learning Controller
BELC	Brain Emotional Learning Controller
CMAC	Cerebellar Model Articulation Controller
FNN	Fuzzy Neural Network
RMSE	Root Mean Square Error
IAE	Integral Absolute Error

5. Conclusion

This paper has presented an FBECMAC-based decoupling controller for a SIMO underactuated nonlinear system within a sliding-mode control framework. Unlike the inconsistent claims in the original draft, the present manuscript verifies the method only by means of a nonlinear ball-beam simulation. The obtained responses show that the proposed controller can stabilize the beam angle and ball position simultaneously, while the auxiliary robust term compensates for the residual approximation error and helps preserve closed-loop stability. Compared with the conventional sliding-mode controller, the proposed method provides improved tracking quality and smoother control action in the reported simulation. Future work should include experimental validation and automatic parameter tuning, for example by particle swarm optimization or other data-driven optimization algorithms.

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